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Different Approaches to Studying the Extreme Properties of Signal Functions Synthesized with Splines

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Abstract The study of parameters of the full energy of signal functions, which are built on the basis of square spline functions, with the use of various evaluation criteria, was conducted. The use of the coefficient of energy concentration (energy criterion) of signals based on quadratic splines is proposed, an analytical equation is obtained for a selective signal whose energy is concentrated near the main lobe, allowing you to choose the optimal values of the width parameter of the transition region and the shape of the spectrum signal implementations. The use of the D-criterion for signals with a limited duration and intersymbol interference is considered. It is shown that the minimization of the level of interference is possible due to the variation of the parameter of the signal spectrum in the transition region, allowing the synthesis of signals with the necessary properties during practical implementation. The use of the criterion of the size of the eye-diagrams allowed us to synthesize signals that are most resistant to shifts of the moments of sampling—jitter. The conditions under which the maximum value of the opening is reached are determined. It was established that in the research of the full energy selective signal evaluation criterion appropriate to use with differential calculus function of several variables, which allows the network in step predetermined design technology to provide the required energy signal indicators. As a result, the use of various criteria for the research of the parameters signals makes it possible at the design stage of signals of mobile communication networks to synthesize such forms that will allow them to physically fully realize them in practice, thereby improving the characteristics of the radio link equipment.

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1 Introduction

The rapid development of mobile technologies in the direction of 4G/5G generation networks is connected, first of all, with development in radio access technologies and an increase in the spectrum bandwidth. The use of third-generation 3G and fourth-generation 4G technologies, such as UMTS and LTE-Advanced. The active development of high-speed Wi-Fi (IEEE 802.11n/ac/ad) implies the use of MIMO-antennas and OFDM/QAM-modulation for which they have used signal functions with minimal intersymbol and inter-channel interference values. As you know, the appearance of intersymbol interference is the result of distortions and fading in the radio interface, therefore, an increase in interference often leads to a decrease in noise immunity and loss. Reducing inter-symbol interference by implementing special designs of signal functions allows you to get the right level of quality characteristics, such as the probability of errors and loss of characters. Besides, the implementation of modern mobile communication technologies is connected with problems of effective use of a frequency resource and a decrease in the level of mutual hindrances. After all, it is from a successful choice of the type of signal structures depends on the capacity of the radio channel, its noise resistance and other characteristics [1, 2].

Saving a radiofrequency resource can be achieved by using spectrally efficient modulation techniques or by using signals with a high concentration of energy in the limited frequency bandwidth. Such signals, according to [1–8], are selective signals with a finite spectrum that satisfy the first Nyquist criterion. Selective signals are quite applicable today, as they allow transmitting information without inter-symbol interference [1–8]. At the same time, studies on the possible improvement of their frequency-time structure are far from completed [1–8].

In studying the parameters of selective signals, various criteria can be used to consider them from different positions: the coefficient of dependence of the energy concentration (energy criterion) of selective signals, the *D*-criterion, the magnitude criterion of the eye-chart and the criterion based on the based on differential computation of functions of many variables [1–4, 9–13].

Studies of signal functions using various criteria are devoted to the work of the authors [1–4, 7, 8], in which various interpolations of spectral characteristics of signals in the frequency-time domain are considered, which best bring them closer to the best designs. According to the work [9], to select the optimal value of the width of the transition region and the shape of the signal spectrum in it, the authors proposed using the criterion of the coefficient of dependence of energy concentration (energy criterion), which allowed us to obtain the signal energy concentrated near the main lobe. For signals with a limited duration and intersymbol interference, in the work [8, 9] was used a *D*-criterion that allowed minimizing the level of interference due to the variation of the signal spectrum parameter in the transition region. At the stage of synthesis of selective signals, special attention in the study in the work

[9] was paid to the criterion of the magnitude of the opening of the eye-diagrams, which allows you to select the signals that are most resistant to shifts in the sampling moments—jitter.

The authors reviewed the criterion using differential computation functions of many variables in the study for the total energy extremum of the signal, which allows providing the required energy signal indicators. In such works [10–18] the authors used spline-functions that allow increasing the accuracy of interpolation while simplifying the procedure of mathematical calculations. In [19–26] it is proposed to use spline-functions to perform a polynomial approximation of signals, which is confirmed by the research results. Similar issues have been considered by the authors in [27–48].

Therefore, the search for new types of interpolation of spectral characteristics of selective signals and the study of their extreme properties of signals in the conditions of active technology development is an important task. In papers [3, 4], the authors developed a method of interpolating the spectral characteristics of the signals under consideration using cubic splines. However, it is interesting enough to study the full energy of the functions of the signal under consideration using square splines, because it will significantly reduce the number of calculations in determining the analytical type of the synthesized signal and its full energy [1–4, 9–13].

It is proposed to investigate the extremal properties of the total energy of signals constructed on the basis of quadratic splines using the methods of differential computation.

The purpose of this work is to use different approaches to the study of the total energy of selective signals, built on the basis of quadratic spline functions. The resulting relationships will allow you to calculate the energy characteristics of the signals and determine the waveform depending on the selected criterion.

2 Methods of Study of the Extreme Properties of Signal Functions Synthesized with Splines

Let's consider a signal with a finite spectrum, which meets the first Nyquist criterion [1–8], for which equality is used in the time domain:

$$g(kT) = \begin{cases} U & \text{for}\, k = 0, \\ 0 & \text{for}\, k = \pm 1, \pm 2, \ldots, \end{cases} \tag{1}$$

where T is the length of a time interval, U is the instantaneous meaning of the pulse count meaning at the time of $k = 0$.

Interpolation selective spectral density of the signal g(t), obtained in [8] is shown in Fig. 1.

As a result, the quadratic spline interpolation [1–9], received signals, the spectral density which has the form

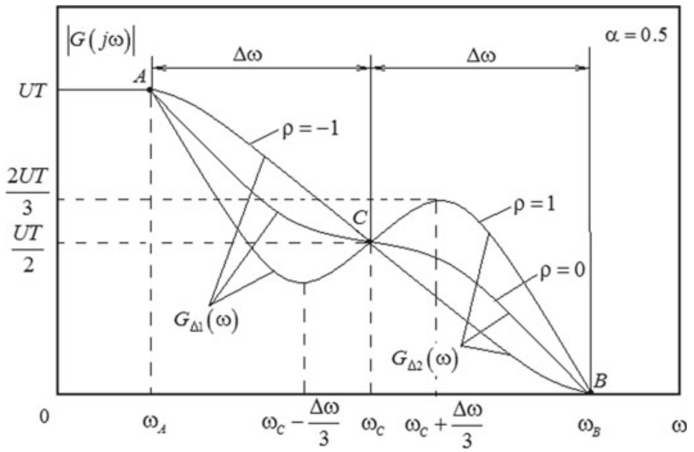

Fig. 1 Interpolation of a selective signal spectrum using a quadratic spline

$$G_{\Delta 2}(\omega) = \frac{UT}{2} + \frac{\rho UT}{\Delta\omega}(\omega - \omega_C) - \frac{UT(1 + 2\rho)}{2(\Delta\omega)^2}(\omega - \omega_C)^2. \tag{2}$$

The analytical expression of the selective signal g(t) obtained in [1–9] is as follows

$$g(t) = 2U\frac{\sin\omega_{c^t}}{\omega_{c^t}}\left[\frac{(1+\rho)\sin(\Delta\omega\, t)}{\Delta\omega\, t} - \frac{(1+2\rho)(1-\cos(\Delta\omega\, t))}{(\Delta\omega)^2 t^2}\right], \tag{3}$$

where U = g(0), ωA = (1 − α)ωC; ωB = (1 + α)ωC; ωC = π/T; α = (ωC − ωA)/ωC is a spectral rounding factor that determines the width of the transition region [ω_A, ω_B] ($0 \le \alpha \le 1$); $\Delta\omega = \alpha\omega_C$.

Figure 2 shows the dependence of the shape of the selective signal in the intervals between equidistant zeros on the parameter ρ.

Therefore, the signals corresponding to the first Nyquist criterion accept the value U at t = 0, and the equidistant points following the interval T have zero meanings. These signals are called selective or countdown signals [1–4, 9, 10]. Their main characteristic is the absence of inter-symbolic interference.

We will investigate the parameters of the full energy of signals construction based on quadratic spline functions using various criteria, namely: the dependence of the concentration of energy (energy criterion) of the selective signal, *D*-criterion, the criterion of the size of the eye-diagram and differential calculus of functions of many variables (method of solving the problem of extremum).

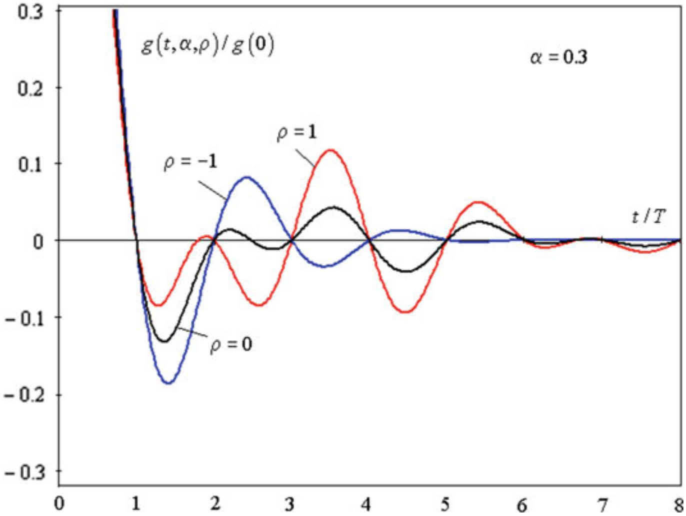

Fig. 2 Dependence of the shape of the selective signal on the parameter ρ

2.1 Dependence of Concentration of Energy of Signal Functions Synthesized by Using of Quadratic Splines

We use the energy concentration dependence coefficient to study the full energy of selective signals. Let us consider the level of concentration of energy of the signal on a given time interval depending on the parameter ρ. As a natural interval let's choose the interval [−nT, nT] between zeros which are symmetrically located concerning the beginning of the coordinate system. It is important to note that when changing the parameter ρ, the position of equidistant zeros of the signal under consideration (3) does not change and inter-symbolic interference does not occur.

The full energy of the selective signal g(t) in accordance with the generalized Reyle formula (Parseval equality) is determined by the relation

$$\int_{-\infty}^{\infty} g^2(t)dt = \frac{1}{2\pi} \int_{-\omega_B}^{\omega_B} |G(j\omega)|^2 d\omega. \tag{4}$$

Partial energy on a segment [−nT, nT] is equal

$$E_t(n, \alpha, \rho) = \int_{-nT}^{nT} g^2(t)dt.$$

The full energy of selective signals g(t) using quadratic splines has the form [9, 10]

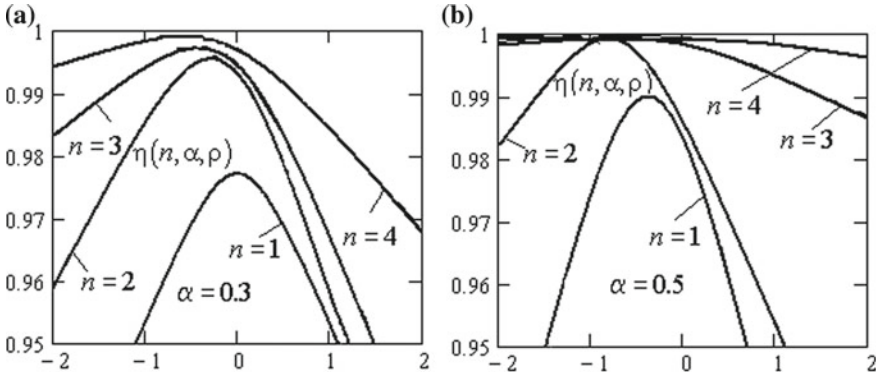

Fig. 3 Dependence of the signal energy concentration $g(t)$ on the coefficient ρ of a quadratic spline $G_{\Delta 2}(\omega_C)$ on a segment [−nT, nT], n = 1, 2 …

$$E_\omega(\alpha, \rho) = U^2 T\left[1 + \frac{\alpha}{30}(2\rho^2 - 3\rho - 12)\right], \tag{5}$$

where ρ is the ratio $G_{\Delta 2}(\omega_C) = \rho\frac{UT}{\Delta\omega}$.

For further research, we use the energy coefficient, which shows how much of the total signal energy g(t) lies on the segment, $[-nT, nT]$, $n = 1, 2 \ldots$ and allows us to investigate the dependence of the signal energy concentration g(t) on the coefficient ρ of a cubic spline $G_{\Delta 2}(\omega_C)$.

$$\eta(n, \alpha, \rho) = E_t(n, \alpha, \rho)/E_\omega(\alpha, \rho), \tag{6}$$

The results of the research are presented in Fig. 3.

The results of the study, which are shown in Fig. 3 show clearly defined extreme whose values increase with n. Optimal values of the corner coefficient are shifted as n changes and are enclosed within $-1 < \rho_{opt} < 0.5$. Thus, by setting ρ_{opt} in (3), we obtain the signal function g(t), which leads to the maximum of the function (6).

The described method—using the criterion of the energy coefficient—allows one to obtain an analytical equation for a signal function whose energy is concentrated near the main lobe. The use of such a signal function allows process simplification its practical implementation at the design stage of mobile systems.

To minimize the intersymbol interference of selective signals, we use the *D*-criterion. Consider the procedure for using the *D*-criterion in the synthesis of selective signals with a minimum level of side lobes. For this purpose, according to [2], we write the equation, which is called the *D*-criterion

$$D(N, \alpha, \rho) = \frac{1}{g(0)}\sum_{k=1}^{N}\left|g\left[\frac{(2k+1)T}{2}\right]\right|. \tag{7}$$

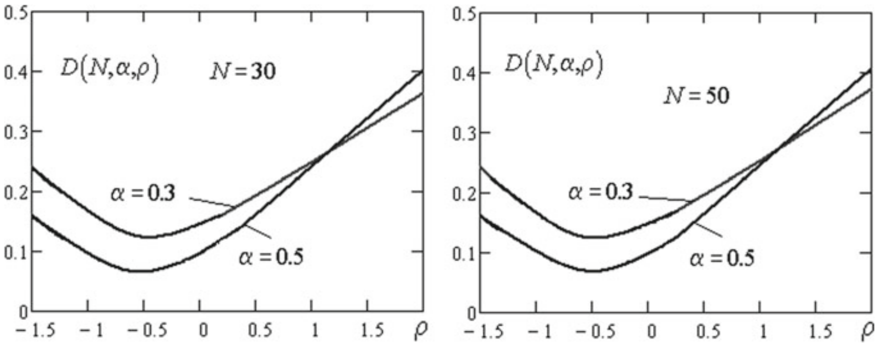

Fig. 4 Minimization of side-lobe levels using *D*-criteria

The decision of the problem is to estimate the value of ρ_{opt} that delivers the minimum of the objective function (7). Given the clear symmetry g(t), one can restrict one-sided calculation of the values of side lobes. Besides, the g(t) signal when $t \to \infty$ damping out quickly, therefore, increasing the number of components in (7) above N = 10 doesn't give a significant increase in accuracy [9, 10]. Figure 4 shows the extreme properties of selective signals using interpolation by quadratic splines relative to criterion (7).

The results of the study allow us to conclude that the use of the D-criterion in the interpolation of the parameters of selective signals allows synthesizing new signal functions, whose extreme properties will improve the characteristics of mobile systems.

2.2 *Using the Eye-Diagram for the Study of Extreme Properties of Signal Functions Constructed Using Quadratic Splines*

One of the criteria for the extreme properties of selective signals is the magnitude of the horizontal disclosure of the eye-diagram, which is often used to assess the to assess quality and quantity characteristics of the noise immunity of digital transmission systems. The eye-chart is a superposition of the signal segments, reduced to a common interval, and reveals the level ISI in the system, and also allows us to estimate the permissible value of random deviations of the registration moments from the optimal position. The reason for the ISI are linear distortion of the amplitude-response (AR) and the phase-response (PR) channel communication range restriction selective jamming [9, 10].

The results of constructing eye-diagram for different values of α and ρ are shown in Figs. 5, 6 and 7.

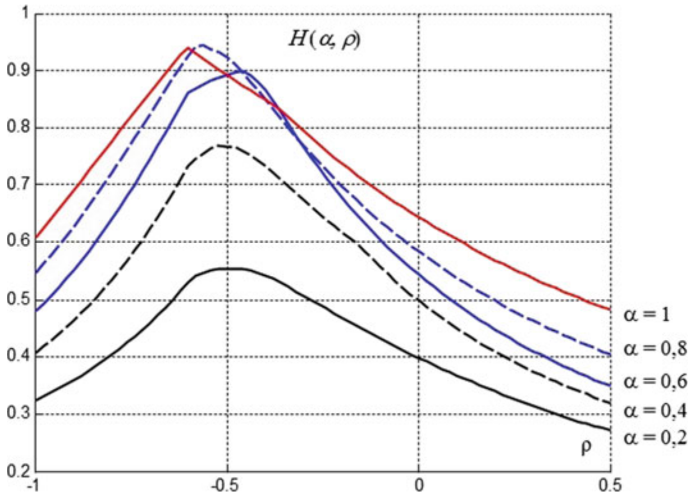

Fig. 5 The dependence of the horizontal disclosure of the eye-diagram on the parameter ρ when the parameter α is changed for a selective signal

Earlier in the works [9–13], the authors analyzed the difference in the openings of eye-charts for selective signals constructed using quadratic splines and signals based on cubic splines, which was 1%. The conditions under which the maximum value of the aperture of the chart is reached are determined. The largest opening is obtained when $\alpha = 1$, $\rho = -0.6$, with $H \approx 0.9582$, which corresponds to the standard values according to the recommendation of ITU-T G.957. Where does the conclusion come from: this slight difference can be ignored, thus using a less calculating aspect, and a more basic practical implementation of the synthesized selective signals built on the quadratic splines.

2.3 Study of the Full Energy of Signal Functions Based on Quadratic Splines Using Differential Computation

Consider the evaluation criterion using the differential computation of functions of many variables, which allows providing the necessary energy indicators of the signal at the design stage of the network of a given technology.

To research the extremum of the full energy of a signal functions from parameters α and ρ, we determine the range of allowed values of parameters α and ρ, the parameter α varies from 0 to 1, and the parameter ρ from $\rho_{min} = -1$ to $\rho_{max} = 1 + \sqrt{2}$. Area of definition of D parameters α and ρ is shown in Fig. 8.

We study the dependence of the full energy E_ω (α,ρ) of the signal functions g(t) on its parameters in the area of $D = \left\{(\alpha, \rho) :\ 0 \le \alpha \le 1, -1 \le \rho \le 1 + \sqrt{2}\right\}$. Find

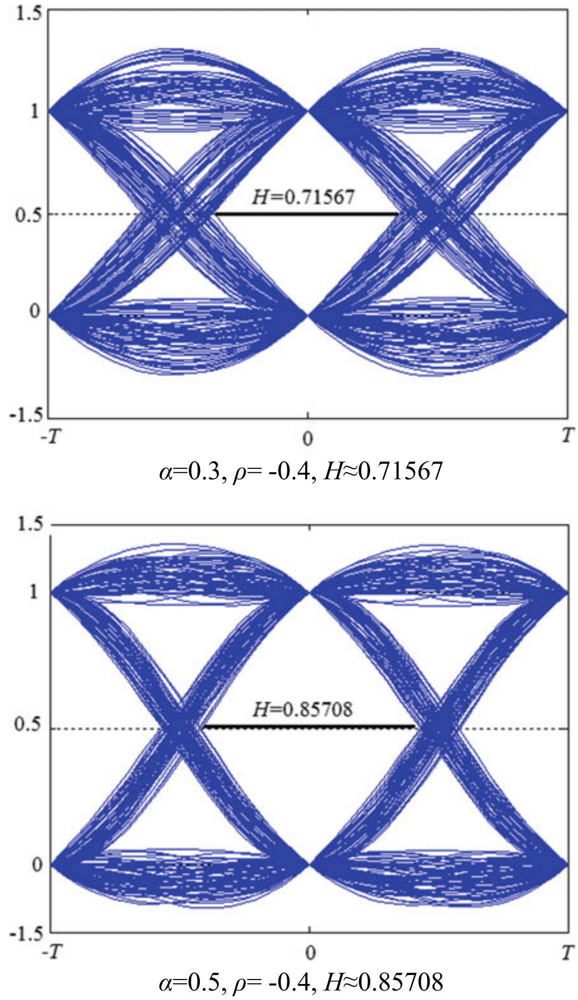

Fig. 6 Eye-diagram of the selective signal g(t)

the minimum and maximum values of the function that are achieved in this area [48–50]. Consider the function.

$$f(\alpha, \rho) = E_{\omega}(\alpha, \rho)/U^2T. \tag{8}$$

Using Eqs. (5) and (8), we get

$$f(\alpha, \rho) = 1 + \frac{\alpha}{30}\left(2\rho^2 - 3\rho - 12\right). \tag{9}$$

To find stationary points of extremum of a function having the form (9), we find its private derivatives.

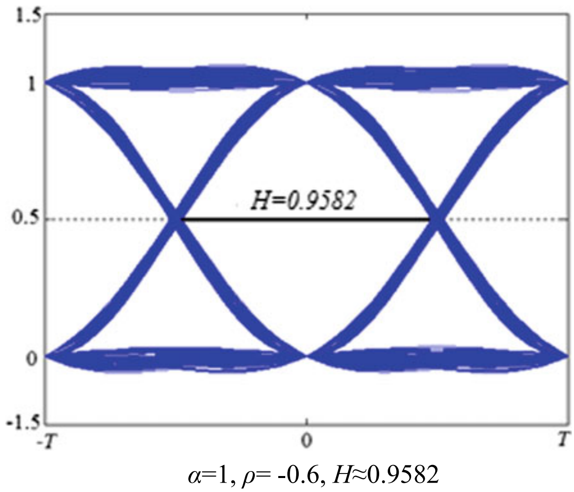

Fig. 7 The maximum possible horizontal opening of the eye-diagram for a signal

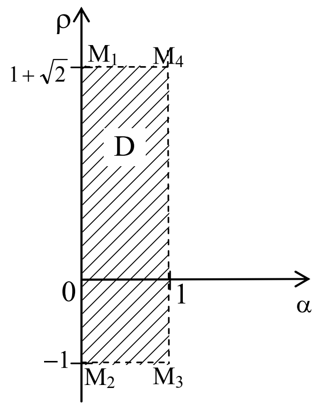

Fig. 8 The area of function $Df(\alpha, \rho)$

$$f'_{\alpha}(\alpha, \rho) = \frac{1}{30}\left(2\rho^2 - 3\rho - 12\right),\ f'_{\rho}(\alpha, \rho) = \frac{\alpha}{30}(4\rho - 3),$$

that exist throughout the area D.

We will find stationary points using the conditions of the extremum of the function of two variables [48–50] by solving the system of equations

$$\begin{cases} \frac{1}{30}(2\rho^2 - 3\rho - 12) = 0, \\ \frac{\alpha}{30}(4\rho - 3) = 0, \end{cases} \Leftrightarrow \begin{cases} 2\rho^2 - 3\rho - 12 = 0, \\ \frac{\alpha}{30}(4\rho - 3) = 0, \end{cases}$$

whence from the first equation $\rho_1 = \frac{3+\sqrt{105}}{4} \approx 3.312$, $\rho_2 = \frac{3-\sqrt{105}}{4} \approx -1.812$, from the second equation $\alpha = 0$ and $\rho = \frac{3}{4}$. From the above, in area D no extremum points.

We find the minimum and maximum meaning of the function $f(\alpha, \rho)$ at the boundary region D. On the segment: M_1M_2 (Fig. 8) we have $\alpha = 0$, $\rho \in [\rho_{min}; \rho_{max}]$. Then function (9) has the form

$$f(\alpha, \rho) = f(\rho) = 1. \tag{10}$$

On the segment M_2M_3, we obtain $\rho_{min} = -1$, $\alpha \in [0, 1]$. In this case, function $f(\alpha, \rho)$ (9) takes the form

$$f(\alpha, \rho) = 1 - \frac{7\alpha}{30}. \tag{11}$$

Having defined $f'_\alpha = -\frac{7}{30}$, we have $f'_\alpha < 0$ for $\alpha \in [0, 1]$. Taking into account the fact that $f(\alpha)$ is a monotonically decreasing function, it takes the minimum value on the interval [0, 1] at $\alpha = 1$ and is equal $f'(1) = \frac{23}{30} \approx 0.77$, and maximum value at $\alpha = 0$ and is equal $f(0) = 1$. In this way,

$$\underset{\alpha \in [0,1]}{f_{max}}(\alpha) = f(0) = 1, \quad \underset{\alpha \in [0,1]}{f_{min}}(\alpha) = f(1) = \frac{23}{30}. \tag{12}$$

On the segment M_3M_4 we have $\alpha = 1$, $\rho \in [\rho_{min}; \rho_{max}]$. The function (9) has have the form

$$f(\alpha, \rho) = f(\rho) = 1 + \frac{1}{30}\left(2\rho^2 - 3\rho - 12\right). \tag{13}$$

Having determined $f'_\rho = \frac{1}{30}(4\rho - 3)$, we get $f'_\rho = 0$ at $\rho = \frac{3}{4}$. As $f'_\rho = \frac{4}{30} > 0$, then $\rho = \frac{3}{4}$ the minimum point for function (13). Accordingly, the minimum value of the function of the form (9) on the segment M_3M_4 has have at the point $M_0\left(1; \frac{3}{4}\right)$ and is equal to

$$f\left(1; \frac{3}{4}\right) = \frac{9}{16} \approx 0.5625 \tag{14}$$

On the segment M_1M_4 we have $\alpha \in [0, 1]$ and $\rho = \rho_{max} = 1 + \sqrt{2}$. Function (9) is represented as

$$f(\alpha, \rho) = f(\alpha, \rho_{max}) = f(\alpha) = 1 + \left(-9 + \sqrt{2}\right)\frac{\alpha}{30}. \tag{15}$$

Calculating the derivative of a function $f(\alpha)$ of the form (15), we obtain

$$f'_\alpha = -\frac{-9 + \sqrt{2}}{30} < 0.$$

Consequently, when $\alpha \in [0, 1]$ the function $f(\alpha)$ monotonously decreases and reaches the maximum value when $\alpha = 0, f_{\max}(\alpha) = 1$, and the minimum value is reached when $\alpha = 1, f_{\min}(\alpha) \approx 0.747$. Thus,

$$\underset{\alpha\in[0,1]}{f_{\max}}(\alpha) = f(0) = 1, \quad \underset{\alpha\in[0,1]}{f_{\min}}(\alpha) = f(1) = \frac{21+\sqrt{2}}{30} \approx 0.747 \tag{16}$$

Comparing the obtained values obtained on different segments M_1M_2 (10), M_2M_3 (12), M_3M_4 (14), M_1M_4 (16), we'll get that the maximum value of the function $f(\alpha, \rho)$ of the form (9) is reached on the segment M_1M_2 and is 1. The smallest value functions of the form (9) is achieved on the interval M_3M_4 has have at the point $M_0\left(1; \frac{3}{4}\right)$ and equals $\frac{9}{16}$. In Fig. 9 there is a view of the function $f(\alpha, \rho)$ in three-dimensional space.

In conclusion, we get

$$f_{\min}(\alpha, \rho) = f_{\min}\left(1; \frac{3}{4}\right) = \frac{9}{16},$$

$$f_{\max}(0, \rho) = 1.$$

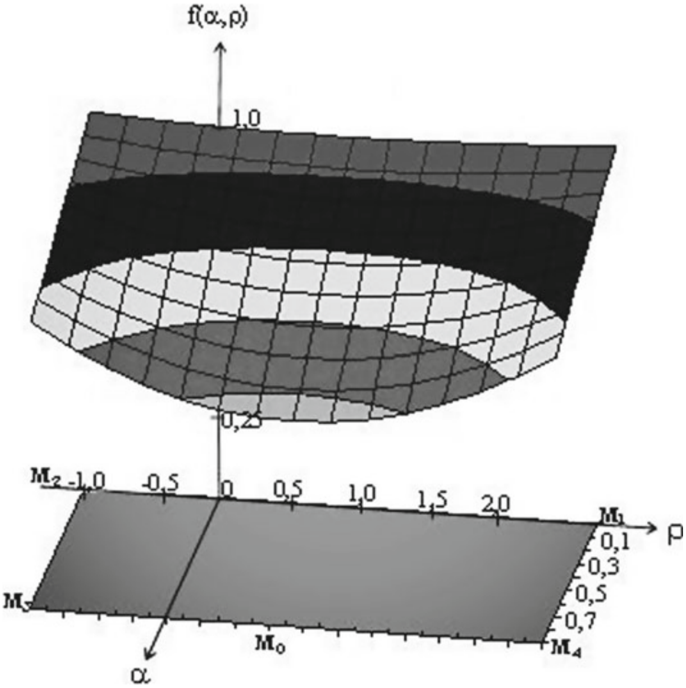

Fig. 9 Considered function $f(\alpha, \rho)$ in three-dimensional space

The conducted research allowed us to conclude that the lowest value, which reaches the full energy of a selective signal using quadratic spline, is equal to

$$E_\omega(\alpha, \rho) = E_\omega\left(1; \frac{3}{4}\right) = \frac{9}{16}U^2T,$$

And maximum

$$E_\omega(\alpha, \rho) = E_\omega(0; \rho) = U^2T.$$

Thus, when selecting an optimal selective signal waveform, infinitely small α values must be used to achieve maximum energy.

3 Conclusion

1. Parameter study performed of the full energy of signal functions using quadratic spline using the following criteria: coefficient of the dependence of energy concentration (energy criterion) of selective signals, *D*-criterion, opening eye-chart criterion and criterion based on the differential computation of functions of many variables.
2. The use of the coefficient of energy concentration (energy criterion) of signal functions using a quadratic spline is proposed. Retrieved parameter relationship width of the transition region and shape of the spectrum in it that when selecting the time interval [−nT, nT] with n = 1 allow obtaining 97% of the maximum possible energy, thus for n = 3, the result can approach sufficiently close to 1 (Fig. 3).
3. When implementing the time characteristics of signal functions using a quadratic spline, in practice it is necessary to minimization the duration of the signals, which will inevitably lead to inter-symbolic interference. It is shown that the minimization of the level of interference is possible due to the variation of the parameter of the signal spectrum in the transition region, allowing the synthesis of signals with the necessary properties during practical implementation. With the optimal choice of the parameters of the width of the transition region and the shape of the spectrum in it, you can receive signals with the desired properties, which will lead to the possibility of practical implementation of these signals.
4. A criterion for the size of the opening of the eye-diagrams for the study of the full energy of signal functions is proposed. The conditions under which the maximum value of the aperture of the chart is reached are determined. The largest opening is obtained at α = 1, ρ = −0.6, while H = 0.9582, which corresponds to the standard values according to the recommendation of ITU-T G.957. It has been substantiated that the use of the criterion of the size of the eye-diagrams allows synthesizing signals that are most resistant to shifts of the moments of sampling—jitter.

5. The parameters of the signal functions were obtained, which allow the energy indicators to be calculated and the optimal waveform to be determined in accordance with the criterion for the differential calculation of functions of many variables using the minimum and maximum values of the function that describes the signal in a given range.
6. The results obtained can be used for the synthesis of new signal functions, when the use of different criteria for the study of signal energy will improve the technical parameters of the radio tract.
7. The obtained results allow asserting that the synthesized selective signals on the basis of quadratic splines have the same properties as those based on cubic splines. In this case, the use of quadratic splines can achieve a significant reduction in the computational process.

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